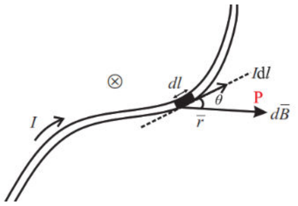
Biot - Savart LAW :

Consider arbitrarily shaped wire carrying a current I. Let dl be a length segment along the wire. The field dB at a distance r from the length segment is

$$dB = \frac{\mu_o}{4\pi}\frac{I\ dl\ sin\theta}{r^2}$$



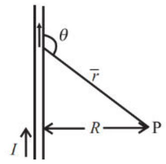
where θ is the angle between $\overline{dl}$ and $\bar{r}$. In vector form ,

$$\overline{dB} = \frac{\mu_o}{4\pi}\frac{I\ \overline{dl}\ X\ \bar{r}}{r^3}$$

Direction of $\overline{dB}$ is given by the cross product of $\overline{dl}\ X\ \bar{r}$

Magnetic field due to a current carrying straight infinite (long) wire

We need to find the magnetic field $\bar{B}$ at a point P which is at a perpendicular distance of R from the wire. Let us consider a small current element $I\ \overline{dl}$ at a distance r from P



$dB = \frac{\mu_o}{4\pi}\frac{I\ dl\ sin\theta}{r^2}$

and t he direction as given by $\overline{dl}\ X\ \bar{r}$ is into the paper.

Assuming symmetry of the upper half and lower half

$B = 2\int_0^\infty dB\ = 2\ \frac{\mu_o}{4\pi}\int_0^\infty \frac{I\ dl\ sin\theta}{r^2}$

BUT $r = \sqrt{l^2+R^2}$ and $\sin(180-\theta) = sin\theta = \frac{R}{r} = \frac{R}{\sqrt{l^2+R^2}}$

Thus, $B = \frac{\mu_o I}{2\pi}\int_0^\infty \frac{dl}{l^2+R^2}\cdot\frac{R}{\sqrt{l^2+R^2}} = \frac{\mu_o IR}{2\pi}\int_0^\infty \frac{dl}{(l^2+R^2)^{\frac{3}{2}}}$

$$B = \frac{\mu_o IR}{2\pi}\frac{1}{R^2} = \boldsymbol{\frac{\mu_o I}{2\pi R}}$$

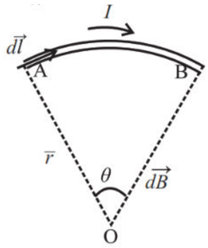
NOTE: If you want to consider for only one half then $B = \frac{\mu_o I}{4\pi R}$

Thus the field is inversely proportional to the distance from the wire

Magnetic field due to a current carrying Circular arc of a wire

Consider a circular arc AB, carrying a current I and subtends an angle θ at the center of the circle O, of which this arc is a part. Let r be the radius. By Biot-Savart law

$$dB = \frac{\mu_o}{4\pi}\frac{I\ dl\ sin\theta}{r^2} = \frac{\mu_o}{4\pi}\frac{I\ dl\ sin90}{r^2} = \frac{\mu_o}{4\pi}\frac{I\ dl}{r^2}$$



The direction is into the plane of the paper at O. Total field at O is

$$B = \int dB = \frac{\mu_o}{4\pi}\int_A^B \frac{I\ dl}{r^2} = \frac{\mu_o}{4\pi}\int_0^\theta \frac{I\ rd\theta}{r^2} == \boldsymbol{\frac{\mu_o}{4\pi}\frac{I\theta}{r}}$$

Where θ is in radians

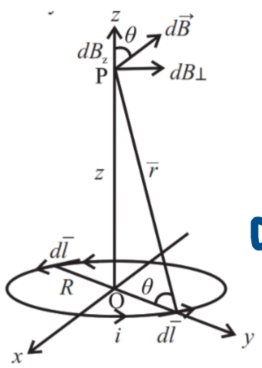
NOTE: For a full circle $B = \frac{\mu_o}{4\pi}\frac{I2\pi}{r} = \frac{\mu_o I}{2r}$

Axial Magnetic field due to a current carrying Circular loop

Consider a circular loop in the xy plane, of radius R and center O, carrying a current I. We need to calculate the magnetic field at point P which is z away from the center and along the z-axis. Let us consider an element $\overline{dl}$ on the loop and P is $\bar{r}$ away from $\overline{dl}$.

$r^2 = R^2 + z^2$

$$dB = \frac{\mu_o}{4\pi}\frac{I\ dl\ sin\theta}{r^2}$$



But θ=90, angle between $\overline{dl}$ and $\bar{r}$

$$Thus, dB = \frac{\mu_o}{4\pi}\frac{I\ dl}{r^2} = \frac{\mu_o}{4\pi}\frac{I\ dl}{R^2+z^2}$$

$\overline{dB}$ is perpendicular to $\overline{dl}$ and $\bar{r}$. Resolving dB into its components $dB_z = dB\ cos\theta$ and $dB_\perp$ along z-axis and perpendicular to z-axis respectively. For all such elements on the ring the sum of all $dB_\perp$ will cancel out. Hence only z components remain

Thus, $B = B_z = \int dB_z = \frac{\mu_o}{4\pi}I\int\frac{dl\ .cos\theta}{Z^2+R^2}$

$\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{Z^2+R^2}}$

$$Thus, B_z = \frac{\mu_o}{4\pi}I\int\frac{R\ dl}{(Z^2+R^2)^{\frac{3}{2}}} = \frac{\mu_o}{4\pi}\frac{IR}{(Z^2+R^2)^{\frac{3}{2}}}.2\pi R$$

$$B_z = \frac{\mu_o}{2}\frac{IR^2}{(Z^2+R^2)^{\frac{3}{2}}}$$

NOTE:

If z=0 (at the center point O), $B = \frac{\mu_o I}{2R}$ and $\frac{\mu_o NI}{2R}$ if there are N turns

For a very large distance ***z >> R***, $B_z = \frac{\mu_o}{2}\frac{IR^2}{z^3} = \frac{\mu_o}{2\pi}\frac{IA}{z^3} = \frac{\mu_o}{2\pi}\frac{m}{z^3}$

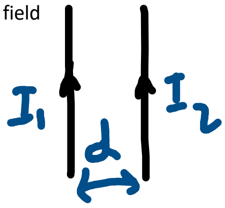
where A=πR² and m=magnetic moment = IA

In vector form $\bar{m} = I\bar{A}$ *and direction of* $\bar{A}$ *is perpendicular to A*

$\overline{B_z} = \frac{\mu_o}{2\pi}\frac{\bar{m}}{z^3}$, Both $\overline{B_z}$ and $\bar{m}$ are perpendicular to the plane of the loop

Force on Two long current carrying parallel wires :

Consider two long parallel straight conductors with currents I_1 and I_2 flowing through them as shown. The magnetic field due to wire 1 on 2 will be



$B_1 = \frac{\mu_o I_1}{2\pi d}$

Force by Lorentz law will be

$F_{21} = B_1 I_2\int dl$

For L length of wire $F_{21} = B_1 I_2 L$

Force per unit length of wire2 $= BI_2 = \boldsymbol{\frac{\mu_o I_1 I_2}{2\pi d}}$

Similarly Force per unit length on wire 1 will be = $\boldsymbol{\frac{\mu_o I_1 I_2}{2\pi d}}$

Both these forces will be attractive in nature.

If the currents were in opposite direction then the magnitude of the forces will be the same but they would be repulsive in nature.

Electric and Magnetic Force

Electric Force F_e = qE where E is the electric field

Magnetic force F_m=qBv sinθ where v is the velocity of the charge q in a magnetic field B and θ is the angle between $\bar{v}$ and $\bar{B}$.

In vector form, $\overline{F_m} = q\ \bar{v}X\bar{B}$

Thus for magnetic force to exist, the charge must be in motion in a magnetic field. The direction of this force is given by Flemings Lefthand rule and is perpendicular to both $\bar{v}$ *and* $\bar{B}$. Since the magnetic force is perpendicular to displacement, hence it ***does no work***. Thus, magnetic force can change the direction of the particle but not its speed.

NOTE: Unit of B is Tesla (T) . 1 T = 10⁴ gauss

EXTRA:

$F_m = qvBsin\theta = It\ v\ Bsin\theta = I\ \frac{L}{v}\ v\ Bsin\theta = BIL\ sin\theta$

$F_m = BIL\ if\ \theta = 90^o, angle\ between\ \bar{v}\ and\ \bar{B}$